# On supergravity domain walls derivable from fake and true superpotentials

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**ABSTRACT:**

We show the constraints that must satisfy the $\mathcal{N}=1$ $\mathcal{D}=4$ SUGRA and the Einstein-scalar field system in order to obtain a correspondence between the equations of motion of both theories. As a consequence, we present two asymmetric BPS domain walls in SUGRA theory associated to holomorphic fake superpotentials with features that have not been reported.

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**RESUMEN:**

Mostramos las restricciones que debe satisfacer SUGRA $\mathcal{N}=1$ $\mathcal{D}=4$ y el sistema acoplado Einstein-campo escalar para obtener una correspondencia entre las ecuaciones de movimiento de ambas teorias. Como consecuencia, presentamos dos paredes dominio BPS asimetricas asociadas a falsos superpotenciales holomorfos con intersantes cualidades.

## I. INTRODUCTION

It is well known in the brane world [1] context that the domain walls play an important role because they allow the confinement of the zero mode of the spectra of gravitons and other matter fields [2], which is phenomenologically very attractive. The domain walls are solutions to the coupled Einstein-scalar field equations [3-7], where the scalar field smoothly interpolates between the minima of the potential with spontaneously broken of discrete symmetry. Recently, it has been reported several domain wall solutions, employing a first order formulation of the equations of motion of coupled Einstein-scalar field system in terms of an auxiliary function or fake superpotential [8-10], which resembles to the true superpotentials that appear in the supersymmetry global theories (SUSY).

Supersymmetry is one of the most promising theoretical concepts in order to build unified models. Thus, it is interesting to find Bogomol'nyi-Prasad-Sommerfield (BPS) configurations [11, 12] such as domain-wall type solutions in this context. In $\mathcal{N}=1$ SUSY theory, exact solutions for a single wall are available [13-15] as well as double wall with two chiral fields in [16]. In $\mathcal{N}=2$ SUSY models, exact solutions of single walls and multi walls have been constructed in [17-19]. The domain walls in local supersymmetry theories or supergravity (SUGRA) have been very difficult to obtain, due to the highly non-linear nature of these theories. However, many attempts performed reveal useful qualitative features of domain walls in SUGRA theory [20-23]. As a result of these attempts, exact domain wall solutions in SUGRA have been found with a smooth limit of weak gravity in $\mathcal{N}=1$ for four dimensions [24] and $\mathcal{N}=2$ in five dimensions [25] .

The extension of SUSY domain walls to SUGRA is not straightforward, because the SUSY vacua change when the theory is coupled to the gravity multiple. This is one of the reasons that prevent us from obtaining the domain wall solutions in SUGRA theory.

In ref. [26] it has been developed a method to embed SUSY theories with domain wall solutions into SUGRA, by introducing a gravitationally deformed superpotential which, after the extension, leaves invariant the SUSY vacua. In spite of the gravitational deformation, the SUGRA theory continues being invariant under SUGRA Kähler transformations as a consequence of the SUSY Kähler transformations. In this paper we will use this gravitational deformation in order to show, under some constraints, there is a correspondence between the BPS equations of the $\mathcal{N}=1$ $\mathcal{D}=4$ SUGRA theory and the first order formulation of the equation of motion of coupled Einstein-scalar field system [8-10]. This correspondence suggests that if one wants to find domain wall solutions in SUGRA, then it is only necessary to solve the first order formulation of coupled Einstein-scalar field equations and verify that the fake superpotential admit an holomorphic representation.

The paper is organized of the following way. In section II we will find the constraints under which the correspondence between $\mathcal{N}=1$ $\mathcal{D}=4$ SUGRA and the coupled Einstein-scalar field system is completed. In section III and IV, we will report two static planar domain wall spacetimes without reflection symmetry in SUGRA theory, which will be found by applying the holomorphic fake superpotentials to the coupled Einstein-scalar field system. Finally, in section V, we will summarize our results.

## II. DOMAIN WALLS IN $\mathcal{D}=4$ $\mathcal{N}=1$ SUPERGRAVITY

Recently in [26] was proposed a gravitational deformation on the superpotential of the $\mathcal{N}=1$ SUSY theory in order to obtain domain wall solutions in the corresponding SUGRA theory. In this section we will consider the gravitational deformation to show that under the most general static metric with planar symmetry and some constraints, the BPS equations are equivalent to the equations of motion of the coupled Einstein-scalar field system.

Consider the bosonic part of the Lagrangian in $\mathcal{D}=4$ where $n$ chiral multiplets $(\phi^i, \chi^i_\alpha)$ are coupled with the gravity multiplet $(e_m{}^{\underline{a}}, \psi_m{}^\alpha)$ given by [26, 27]

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$$e^{-1}\mathcal{L} = -\frac{1}{2\kappa^2}R - K_{ij*}g^{mn}\partial_m\phi^i\partial_n\phi^{j*} - V_{lc}(\phi), \tag{1}$$

where

$$V_{lc} = e^{\kappa^2 K}(K^{ij*}D_iW_{lc}D_{j*}W^*_{lc} - 3\kappa^2|W_{lc}|^2), \tag{2}$$

$$D_iW_{lc} = \partial_iW_{lc} + \kappa^2W_{lc}\partial_iK. \tag{3}$$

In the Lagrangian, $K_{ij*} \equiv \partial_{ij*}K$ with $K$ being the Kähler potential $K \equiv K(\phi, \phi^*)$. $W_{lc}$ is an holomorphic function called superpotential, $\kappa$ is the gravitational coupling constant, $g_{mn}$ is the metric of the spacetime and $e$ is the determinant of the vierbein $e_m{}^{\underline{a}}$. The local Lorentz vector indices are denoted by underlined letters such as $\underline{a}$ and the vector indices transforming under general coordinate transformations are denoted by Latin letters such as $m, n = 0, 1, 2, 3$. The left (right) handed spinor indices are denoted by undotted (dotted) Greek letters such as $\alpha$ ($\dot{\alpha}$).

We are interested in bosonic solutions since these are the ones that correspond to classical solutions. A bosonic solution will be supersymmetric if the supersymmetry transformations vanish for some infinitesimal supersymmetric parameter $\zeta(y)$. In the absence of fermion fields, the bosonic fields always are invariants, and it is necessary only that the supersymmetry transformations of the fermion fields vanish, thus

$$\delta_\zeta\psi_m \;=\; 2\kappa^{-1}D_m\zeta + i\kappa e^{\frac{\kappa^2}{2}K}W_{lc}\sigma_m\bar{\zeta} = 0, \tag{4}$$

$$\delta_\zeta\chi^i \;=\; i\sqrt{2}\sigma^m\bar{\zeta}\partial_m\phi^i - \sqrt{2}e^{\frac{\kappa^2}{2}K}K^{ij*}D_{j*}W^*_{lc}\zeta = 0, \tag{5}$$

where

$$D_m\zeta = \partial_m\zeta + \zeta\omega_m + \frac{i\kappa^2}{2}\sum_i \Im\left[\partial_iK\partial_m\phi^i\right]\zeta, \qquad (\zeta\omega_m)_\alpha = \frac{1}{2}\omega_{m\underline{ab}}(\sigma^{\underline{ab}})_\alpha{}^\beta\zeta_\beta \tag{6}$$

and $\omega_{m\underline{ab}}$ is the spin connection.

The BPS equations can be derived from the half supersymmetric condition where we have redefined the conserved supersymmetry parameter which only depends on one extra coordinate $x^3 = y$

$$\zeta(y) = e^{i\theta(y)}\sigma^{\underline{2}}\bar{\zeta}(y). \tag{7}$$

Next, let us consider that $\phi^i = \phi^i(y)$ and the following metric ansatz

$$g_{mn} = e^{2A(y)}(-dt_mdt_n + dx^k_mdx^k_n) + e^{2H(y)}dy_mdy_n. \tag{8}$$

Thus, from $m = 0, 1, 2$ in the supersymmetry transformation (4) we obtain the first order equation for the warp factor (prime indicate differentiation with respect to $y$)

$$e^{-H}A' = -i\kappa^2e^{-i\theta}e^{\frac{\kappa^2}{2}K}W_{lc}. \tag{9}$$

On the other hand, from equation (5) we obtain first order equation for the matter fields

$$e^{-H}(\phi^i)' = -ie^{i\theta}e^{\frac{\kappa^2}{2}K}K^{ij*}D_{j*}W^*_{lc}. \tag{10}$$

Rewriting the half supersymmetric condition as $\zeta_\alpha = e^{\frac{i}{2}(\theta+\frac{\pi}{2})}|\zeta_\alpha|$, and substituting it into equation (4) for $m = 3$, we find

$$|\zeta'_\alpha| = \frac{A'}{2}|\zeta_\alpha|, \qquad \theta' = -\kappa^2\Im\left[\sum_i(\phi^i)'\partial_iK\right]. \tag{11}$$

Equations (9), (10) and (11) are collectively called BPS equations.

Now, we want to rewrite the BPS equations in a way that resembles the first order formulation of coupled Einstein-scalar field system [8–10]. For this, let us consider the gravitational deformation of the superpotential proposed in [26]

$$W_{lc}(\phi) = e^{-\frac{\kappa^2}{2}\tilde{K}(\phi)}\,\tilde{W}_{gl}(\phi), \tag{12}$$

where $\tilde{K}(\phi) \equiv K(\phi, \phi^* \to \phi)$ and $\tilde{W}_{gl} = W_{gl} + a$, with $W_{gl}$ being the superpotential in SUSY theory (non-linear sigma model) and $a$ an arbitrary constant.

Observe that the gravitational deformation (12) obliges us to distinguish between the superpotentials of the local ($W_{lc}$) and global ($W_{gl}$) supersymmetric theory, unlike to the usual approach where both superpotentials are the same [21, 28]. However, the SUGRA theory described by the Lagrangian (1) under (12) continues being invariant under SUGRA Kähler transformations, maintain a smooth limit of vanishing gravitational coupling, and moreover, preserves the SUSY vacua, because the critical points of $\tilde{W}_{gl}$ are the same critical points of $W_{lc}$, as was demonstrated in [26].

Substituting the deformation (12) into equation (10) we obtain

$$e^{-H}(\phi^i)' = -ie^{i\theta}e^{\frac{\kappa^2}{2}(K-\tilde{K}^*)}K^{ij*}\left[\kappa^2\,\tilde{W}_{gl}\partial_{j*}\left(K - \frac{1}{2}\tilde{K}^*\right) + \partial_{j*}\,\tilde{W}^*_{gl}\right]. \tag{13}$$

Now, if we choose real scalar fields and we demand the following constraint be satisfied

$$\partial_{j*}\left(K - \frac{1}{2}\tilde{K}^*\right)\bigg|_{\phi\,\in\,\Re} = 0, \tag{14}$$

we find that equation (10) changes to

$$e^{-H}(\phi^i)' = K^{ij}\partial_j\,\tilde{W}_{gl}, \tag{15}$$

and hence the warp factor equation (9) and the scalar potential take the form

$$e^{-H}A' = -\kappa^2\,\bar{W}_{gl}, \tag{16}$$

$$V_{lc} = K^{ij}\partial_i\bar{W}_{gl}\partial_j\bar{W}_{gl} - 3\kappa^2\bar{W}_{gl}^2, \tag{17}$$

where we have taken $\theta = \pi/2$ as a solution to (11). Expression (11) determines the Killing spinors which have two real Grassmann parameters $\epsilon_1, \epsilon_2$ corresponding to the two conserved supersymmetric directions on the BPS solutions

$$\zeta = -e^{\frac{A}{2}} \times \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}. \tag{18}$$

The constraint (14) strongly conditions the Kähler potential that may admit the theory to simplify the BPS equations. In fact, a Kähler potential which satisfies this constraint and reduces (15) and (16) to the equations of motion of the coupled Einstein-scalar field system, is the minimal Kähler potential $K = \phi^i\phi^{i*}$. Another interesting issue to worth mentioning is that (15) and (16) suggest that if one wants to obtain domain wall solutions associated to a Kähler potential different to the minimal, it is necessary to find a suitable combination of the $K$ and $\bar{W}_{gl}$.

Therefore, if we consider that SUGRA theory is compatible with:

1. the minimal Kähler potential and only one chiral multiplet,
2. the gravitational deformation (12) and the constraint (14),

then we obtain the first order motion equations of the coupled Einstein-scalar field system

$$e^{-H}\phi' = \frac{d}{d\phi}\bar{W}_{gl}, \tag{19}$$

$$e^{-H}A' = -\kappa^2\,\bar{W}_{gl} \tag{20}$$

and

$$V_{lc} = \left(\frac{d}{d\phi}\bar{W}_{gl}\right)^2 - 3\kappa^2\bar{W}_{gl}^2. \tag{21}$$

The coupled Einstein-scalar field system is a non-supersymetric theory, so in this context $\bar{W}_{gl}$ is a fake superpotential and is not subject to holomorphic restrictions [5, 8–10, 29, 30] . Remember that the SUSY theory requires that a true superpotential be holomorphic. Therefore, the development exposed here shows that if the fake superpotential with $\phi$ complex is an holomorphic function, then the solutions obtained to the coupling are also solutions in $\mathcal{N} = 1$ $\mathcal{D} = 4$ SUGRA theory with the minimal Kähler potential.

Next, briefly we will revise two examples reported in [21, 24, 26] in order to illustrate the approach presented

## A. From the sine-Gordon superpotential

Let us consider the fake superpotential $\bar{W}_{gl}$ given by

$$\bar{W}_{gl}(\phi) = \beta\sin\frac{\phi}{\sqrt{\delta}} + a, \tag{22}$$

where $\beta$ and $\delta$ are real constants and $\delta > 0$. From (19, 20, 21) with $H = 0$ we obtain

$$\phi(y) = \sqrt{\delta}\arctan\sinh\frac{\beta y}{\delta}, \qquad A(y) = -\kappa^2\left[a\,y + \delta\ln\cosh\frac{\beta y}{\delta}\right] \tag{23}$$

and

$$V_{lc}(\phi) = \beta^2\left(\frac{1}{\delta} + 3\kappa^2\right)\cos^2\frac{\phi}{\sqrt{\delta}} - 3\kappa^2\left[a^2 + \beta^2 + 2a\beta\sin\frac{\phi}{\sqrt{d}}\right]. \tag{24}$$

These solutions represent a three-parameter family of plane symmetric static domain wall spacetimes, whose reflection symmetry along the direction perpendicular to wall depends on the $a$ parameter. In fact, for $a = 0$ it is obtained the symmetric solution, being asymptotically (i.e. far away from the wall) $AdS$ with cosmological constant $\Lambda = -3\beta^2\kappa^2$ and where $\delta$ plays the role of the wall's thickness. It has been shown that this smooth domain wall geometry (parameterized in a slightly different form) localizes gravity on the wall in [22] and has as the distributional $\delta \to 0$ thin wall limit [5, 31] the Randall-Sundrum scenario [1] in the sense of [32].

The superpotential (22) is an holomorphic function for $\phi$ complex, which is key to guarantee that (23,24) be supersymmetric solutions. Moreover (22) turns out to be the superpotential of the SUSY sine-Gordon model with the minimal Kähler potential [24].

## B. From the double-well superpotential

Now, let us consider the fake superpotential given by

$$\bar{W}_{gl}(\phi) = \beta\frac{\phi}{\sqrt{\delta}}\left(1 - \frac{1}{3}\frac{\phi^2}{\delta}\right) + a, \tag{25}$$

where the parameters satisfiy the same conditions of the last example. Thus, from the (19, 20, 21) with $H = 0$ we find

$$\phi(y) = \sqrt{\delta}\tanh\frac{\beta\,y}{\delta}, \qquad A(y) = -\kappa^2\left[a\,y + \frac{1}{3}\delta\left(2\ln\cosh\frac{\beta\,y}{\delta} + \frac{1}{2}\tanh^2\frac{\beta\,y}{\delta}\right)\right] \tag{26}$$

and

$$V_{lc}(\phi) = \frac{\beta^2}{\delta}\left(1-\frac{\phi^2}{\delta}\right)^2 - 3\kappa^2\left[\beta\frac{\phi}{\sqrt{\delta}}\left(1-\frac{1}{3}\frac{\phi^2}{\delta}\right)+a\right]^2. \tag{27}$$

These solutions also represent a three-parameter family of plane symmetry static domain wall spacetimes with reflection symmetry dependent on the $a$ parameter, such that for $a = 0$, this spacetime is asymptotically $AdS$ with cosmological constant $\Lambda = -4/3\beta^2\kappa^2$. This geometry, parameterized in a different form, was proposed in [10] as a smooth realization of the scenario of [1] and has been shown to localize gravity on the wall in [33].

The expression (25) is the simplest superpotential which gives rise to two isolated supersymmetric vacua in SUSY theories and with which is possible to construct the most general supersymmetric renormalisable lagrangian [27, 34, 35].

Observe that in the two examples presented in this section the $\tilde{W}_{gl}$ are the associated to SUSY theories reported in the literature, which immediately allow us to obtain from the correspondence previously discussed, the domain wall SUGRA, as well as the extension of the associated SUSY theory to SUGRA model, in agreement with [26].

Next we will show two domain wall solutions embedded into SUGRA, associated to $\tilde{W}_{gl}$ that have not been identified in none of the SUSY theories with minimal Kähler potential formulated so far.

## III. ASYMMETRIC BRANE WORLD

Consider the following fake superpotential reported in [30]

$$\tilde{W}_{gl}(\phi) = \frac{\beta}{4}\frac{\phi^2}{\delta}\left(1-\ln\frac{\phi^2}{\delta}\right)+a, \qquad a \equiv -\frac{\alpha}{4}. \tag{28}$$

Now, from the (19, 20, 21) with $H = 0$ we find

$$A(y) = -\frac{\kappa^2}{4}[-\alpha\, y + \delta\exp(-2\exp(-\beta y/\delta)) - \delta\,\mathrm{Ei}(-2\exp(-\beta y/\delta))], \tag{29}$$

with Ei the exponential integral given by

$$\mathrm{Ei}(u) \equiv -\int_{-u}^{\infty} d\tau\frac{e^{-\tau}}{\tau} \tag{30}$$

and where $\alpha, \beta, \delta$ are real constants with $\delta$ and $\beta$ positive definite

$$\phi(y) = \sqrt{\delta}\exp(-\exp(-\beta y/\delta)) \tag{31}$$

and

$$V_{lc}(\phi) = \frac{1}{4}\left\{\beta^2\frac{\phi^2}{\delta^2}\ln^2\frac{\phi^2}{\delta} - \frac{3}{4}\kappa^2\left[\beta\frac{\phi^2}{\delta}\left(1-\ln\frac{\phi^2}{\delta}\right)-\alpha\right]^2\right\}, \tag{32}$$

where the field interpolates between the two non-degenerate minima (see Fig.1) of $V_{lc}$, $\phi_1 = 0$ and $\phi_2 = \sqrt{\delta}$. This represents a three-parameter family of plane symmetry static domain wall spacetime without reflection symmetry along the direction perpendicular to the wall. In this point it is worthwhile to stand out that this asymmetry doesn't depend on the $a$ parameter, contrary to the previous examples.


Figura 1: Plots of the asymmetric kink $\phi(y)$ (left) and the energy density $\rho(y)$ (right).

That is to say, for any value of $a$ the solution is always asymmetric. Therefore, the asymmetry is intrinsic to the domain wall.

If $\beta > \alpha > 0$, the spacetime is asymptotically $AdS$ with cosmological constant $-3\alpha^2\kappa^2/16$ for $y < 0$, and $-3(\beta-\alpha)^2\kappa^2/16$ for $y > 0$. In fact, following [5], the distributional $\delta \to 0$ thin wall limit of this geometry has been obtained in [30], showing that this spacetime behave asymptotically [31] as an $AdS$ spacetime with different cosmological constants at each side of the wall. This behavior also can be observed in the profile of the energy density, which asymptotically tends to these values of the cosmological constant - see Fig. 1. On the other hand, for this interval of $\alpha$, the massless zero mode of graviton is normalizable [30], which characterize the interval as a confinement region.

If $\alpha = \beta$ ($\alpha = 0$), then for $y \to +\infty$, the spacetime is asymptotically flat ($AdS$) with a warp factor increasing toward a horizontal asymptote (decreasing), and for $y \to -\infty$ the spacetime is asymptotically $AdS$ (flat), with a warp factor exponentially decreasing (increasing toward horizontal asymptote). On the other hand, if $0 > \alpha > \beta$, then the spacetime for $y \to \pm\infty$ is asymptotically $AdS$. For $\alpha > \beta$ ($\alpha < 0$), the warp factor increasing (decreasing) for $y \to +\infty$ and decreasing (increasing) for $y \to -\infty$. In Fig. 2 and Fig. 3 it is possible to see this behavior for $e^A$, $\tilde{W}_{gl}$ and $V_{lc}$ in each case.

These solutions were obtained in the frame of a non-supersymmetric theory, coupled Einstein -scalar field system. However, the fake superpotential (28) is an holomorphic function for $\phi$ complex, satisfying the only necessary requirement for it to be a true superpotential of a SUSY theory. Therefore, in accordance with section II and [26], the equations (29,31,32) are domain wall solutions to the SUGRA theory that emerges when the SUSY theory associate to the superpotential (28) is coupled with the gravity multiplet.

Remarkably (28) resembles the superpotential present in the effective $\mathcal{N} = 1$ super Yang-Mills theory known as Veneziano Yankielowicz theory [36, 37], where the Kähler potential is not minimal but $K \sim (\phi\phi^*)^{1/3}$ with $\phi$ the gluino condensate. This Kähler potential satisfies the constraint (14) however, it nether allows to establish the correspondence nor to find the exact domain wall solutions.

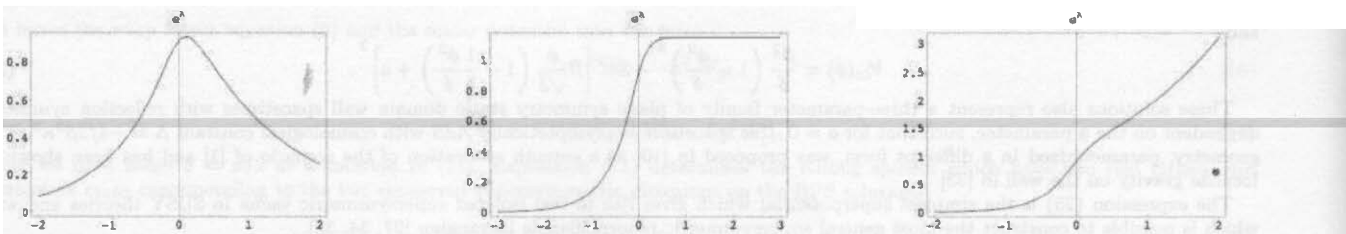

Figura 2: Plots of the warp factor for $\alpha < \beta$ (left), $\alpha = 0, \beta$ (center) and $0 > \alpha > \beta$ (right).

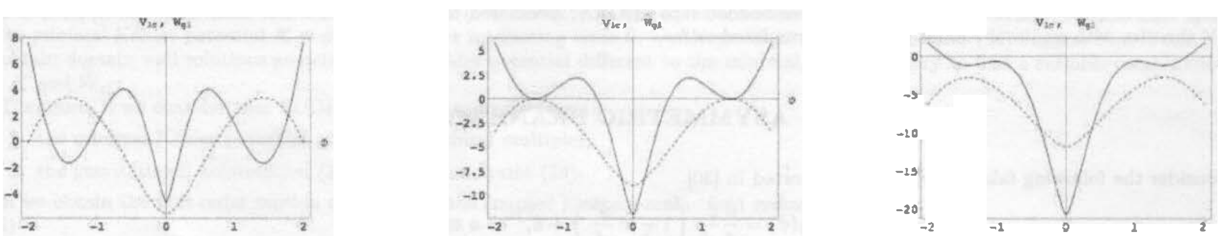

Figura 3: Plots of the scalar potential (continuos line) and superpotential (dashed line) for $\alpha < \beta$ (left), $\alpha = \beta$ (center) and $0 > \alpha > \beta$ (right).

## IV. DOUBLE BRANE WORLD

Let us consider the following fake superpotential

$$\tilde{W}_{gl}(\phi) = \beta(\sin\phi/\phi_0)^{\frac{2s-1}{s}} + a, \qquad \phi_0 = \frac{\sqrt{\delta(2s-1)}}{s}, \qquad -\pi\phi_0/2 \leq \phi \leq \pi\phi_0/2 \tag{33}$$

and

$$H(y) = -\frac{1}{2s}\ln[1 + (\beta y/\delta)^{2s}]. \tag{34}$$

Then, from (19, 20, 21) we obtain

$$A(y) = \frac{1}{2}\kappa^2 y\left\{-2a\,{}_2F_1[l, m, n, -(\beta y/\delta)^{2s}] - \frac{\delta}{ys}\ln[1 + (\beta y/\delta)^{2s}]\right\}, \tag{35}$$

$$\phi(y) = \phi_0 \arctan(\beta y/\delta)^s, \tag{36}$$

$$V_{lc}(\phi) = \beta^2\delta(1-2s)\cos^2(\phi/\phi_0)\sin^{2(s-1)/s}(\phi/\phi_0) - 3\kappa^2\left[a + \beta\sin^{(2s-1)/s}(\phi/\phi_0)\right]^2, \tag{37}$$

where ${}_2F_1$ is the hypergeometric function with $l = m = 1/(2s)$; $n = 1 + l$ and $\beta, \delta, a$ and $s$ are real constants, with $s$ being an odd integer in order for the field (36) to be a double kink (see Fig. 4) interpolating between the minimal of the scalar potential (37), $\phi(y \to \pm\infty) = \pm\phi_0\pi/2$. For these values of $s$ , this is a four-parameter family of plane symmetric static double domain wall spacetime without reflection symmetry along the direction perpendicular to the wall. Contrary to the previous case, the asymmetry of this solution is not intrinsic but depends of the $a$ parameter. In fact, for $a = 0$ one obtains the reflection symmetry static double domain wall reported in [6].

Figura 4: Plots of the double kink $\phi(y)$ (left) and the energy density $\rho(y)$ (right) for $s = 3, 5$ and 7. The thickness of the line increases with increasing $s$.

If $|a| < \beta$, the spacetime is asymptotically $AdS$ with cosmological constant $-3\kappa^2(\beta - a)^2$ for $y < 0$, and $-3\kappa^2(\beta + a)^2$ for $y > 0$. This behavior also can be observed in the profile of the energy density, which is peaked around two values and asymptotically tends to different cosmological constants - seen Fig. 4. Moreover, in this interval the warp factor (35) is a bounded function which assures the confinement of the mode zero of graviton.

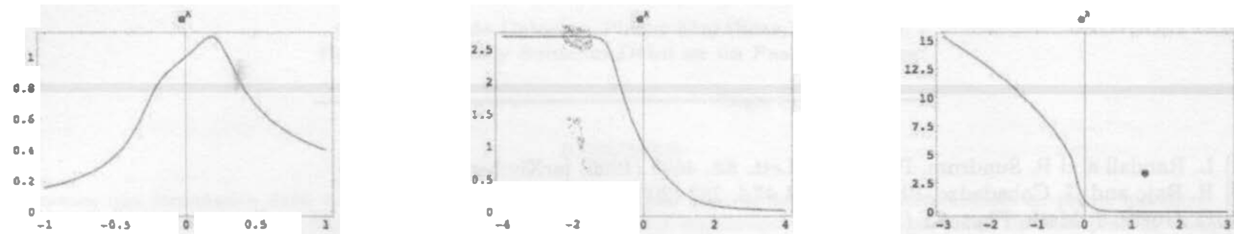

Figura 5: Plots of the warp factor for $|a| < \beta$ (left), $a = \pm\beta$ (center) and $|a| > \beta$ (right).

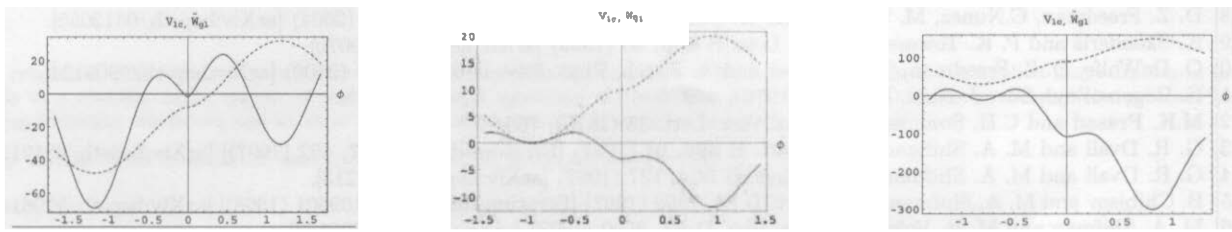

Figura 6: Plots of the scalar potential (continuos line) and superpotential (dashed line) for $|a| < \beta$ (left), $a = \pm\beta$ (center) and $|a| > \beta$ (right).

If $a = \beta(-\beta)$, then for $y \to +\infty$ the spacetime is asymptotically $AdS$ (flat) and for $y \to -\infty$ is asymptotically flat ($AdS$). In the case, where the spacetime is asymptotically $AdS$, the warp factor decreases exponentially. If $a > \beta(a < -\beta)$, then the spacetime for $y \to +\infty(-\infty)$ is asymptotically $AdS$ and the warp factor decreases and for $y \to -\infty(+\infty)$ increases, as seen in Fig. 5. On the other hand, Fig. 6 show the graphics of $\bar{W}_{gl}$ and $V_{lc}$ are compatible with these intervals.

The $\bar{W}_{gl}$ can be considered as a generalization of sine-Gordon superpotential (22). In fact this generalized superpotential, as in the particular sine-Gordon case, is holomorphic for $\phi$ complex and therefore is a legitimate superpotential, in spite of not having been considered in some SUSY theories yet. Hence, the equations (35,36,37) are solutions to the SUGRA theory that emerges when the SUSY theory associated to superpotential (33) is coupled to gravity.

## V. SUMMARY AND OUTLOOK

We have found the conditions that the SUGRA theory must satisfy under the gravitational deformation proposed in [26], so that the equations that give solution to the non-supersymmetric coupled Einstein-scalar field system in terms of a fake superpotential [8–10] be understood as the BPS equations of the SUGRA theory in question. The fundamental conditions are i) that the Kähler potential satisfy the constraint (14) and ii) that the fake superpotential for $\phi$ complex is an holomorphic function. The latter assures a smooth limit of vanishing gravitational coupling. In principle, a Kähler potential which allows us to find the desired equivalence is the minimal. In the context of domain wall solutions in SUGRA , to choose a Kähler potential different from the minimal, implies to think about an appropriate combination of Kähler potential and fake superpotential.

In order to generate new static domain wall solutions to SUGRA theory, we proposed two holomorphic functions as the fake superpotentials associated to the equations of the coupled Einstein-scalar field system. These solutions are related to $AdS$ static spacetime without reflection symmetry along the direction perpendicular to the wall. The first solution is compatible with a kink-like scalar field and the second to a double kink. In the first case, the asymmetry is intrinsic to the domain wall spacetime, because the kink interpolates between two non-degenerate minima of $Z_2$ symmetric potential. On the contrary, in the second case, where the asymmetry is a product of the additional $a$ parameter that was introduced with the purpose of observing gravitational deformations in the model. In fact, when $a = 0$ it can be recovered the solution with reflection symmetry.

Since the presented fake superpotentials admit an holomorphic representation, we can say that they are legitimate superpotentials of a SUSY theory, and thus the obtained static asymmetric domain wall geometries turn out to be BPS solutions to the SUGRA theory corresponding to each superpotential with a minimal kinetic term.

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